M.Sc. - I (Mathematics) (NEP Pattern) Semester-II

Major DSC-1 - Field theory

P. Pages : 2 **GUG/S/25/15393**

Time : Three Hours Max. Marks : 80

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that Every PID is a UFD, but a UFD is not necessarily a PID. **8**

b) Prove that an irreducible element in a commutative principal ideal domain (PID) is always prime. **8**

OR

c) Prove that If $f(x), g(x) \in R[x]$ then $c(fg) = c(f)c(g)$. In particular, the product of two primitive polynomial is primitive. **8**

d) Prove that Every Euclidean domain is a PID. **8**

UNIT – II

2. a) Let $f(x) \in z[x]$ be primitive. Then prove that $f(x)$ is reducible over Q if and only If f(x) reducible over Z. **8**

b) Let $F \subseteq E \subseteq K$ be fields. If $[K:E] < \infty$ and $[E:F] < \infty$ **8**
Then prove that
i) $[K:F] < \infty$
ii) $[K:F] = [K:E]\,[E:F]$

OR

c) Let $f(x) = a_0 + a_1 x + --- + a_{n-1} x^{n-1} + x^n \in Z[x]$ be a monic polynomial. If f(x) has a root $a \in Q$ then prove that $a \in Z$ and a / a_0. **8**

d) Let p(x) be an irreducible polynomial in f[x]. Then prove that There exist an extension E of F in which p(x) has a root. **8**

UNIT – III

3. a) Let K be a splitting field of the polynomial $f(x) \in F[x]$ over a field F. If E is another splitting field of f(x) over F. Then prove that there exists an isomorphism $\sigma : E \to K$ that is Identity on F. **8**

b) Let F = Z/(2). The splitting field of $x^3+x^2+1\in F[x]$ is a finite field with eight elements. **8**

OR

c) The splitting field of $f(x)=x^4-2\in Q[x]$ over Q is $Q\left(2^{1/4},i\right)$ and its degree of extension is 8. **8**

d) If $f(x)\in F[x]$ is irreducible over F. Then prove that roots of f(x) have the same multiplicity. **8**

UNIT – IV

4. a) State and prove fundamental theorem of Galois theory. **8**

b) The Galois group of $x^3-2\in Q[x]$ is the group of symmetries of the triangle. **8**

OR

c) The Galois group of $x^4-1\in Q[x]$ is the Klein four group. **8**

d) State and prove fundamental theorem of Algebra. **8**

5. a) Define **4**

i) Irreducible element.

ii) Principal Ideal Domain.

b) Find the minimal polynomials over Q of the following numbers. **4**

i) $\sqrt{2}+5$

ii) $3\sqrt{2}+5$

c) Define **4**

i) Separable extension.

ii) Simple extension.

d) Define **4**

i) Galois group

ii) Galois extension

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